DIMENSION OF VECTOR SP.

Let $V(\mathbb{F})$ be a V.S.

$\dim(V) = |B| = \text{card}(B) =$ no. of elements in B

(1) $V = P_n(\mathbb{R})(\mathbb{R})$

$\dim(V) = n+1$, $B = \{1, x, x^2, x^3, \ldots, x^n\}$

(2) $V = M_{m\times n}(\mathbb{R})(\mathbb{R})$

$\dim V = mn$, $B = \{e_{11}, e_{12}, \ldots, e_{1n}, e_{21}, e_{22}, \ldots e_{mn}\}$

(3) $V = \mathbb{F}^n(\mathbb{F})$

$\dim(V) = n$

(4) $V = \mathbb{F}(\mathbb{F})$

$\dim V = 1$, $B = \{1\}$, $\mathbb{R}(\mathbb{R})$, $\mathbb{C}(\mathbb{C})$, $\mathbb{Q}(\mathbb{Q})$ etc

(5) $V(\mathbb{F}) = \mathbb{C}(\mathbb{R})$

$\dim V = 2$, $B = \{1, i\}$

(6) $V(\mathbb{F}) = \mathbb{C}(\mathbb{Q})$

$\dim V = \infty$

(7) Let $\mathbb{F}$ be a field and K be a subfield of $\mathbb{F}$. and $\mathbb{F}(K)$ is always a V.S.

$\dim(F(K))$ depends on $\mathbb{F}$ & K

$\dim(\mathbb{C}(\mathbb{R})) = 2$, $\dim(R(\mathbb{Q})) = \infty$, $\dim(\mathbb{Q}\sqrt{2}(\mathbb{Q}\sqrt{2})) \to 1$

(8) Let $V = M_{n\times n}(\mathbb{F})(K)$ be a V.S. over the field K, when $K \subset \mathbb{F}$.

$\dim(M_{n\times n}(\mathbb{F}))(K) = n^2 \dim(\mathbb{F}(K))$

Eg: $V = M_{2\times 2}(\mathbb{C})(\mathbb{R})$; $\dim V = 2^2 \times 2$

$= 8$